


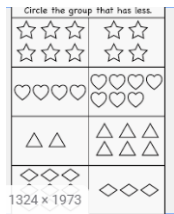




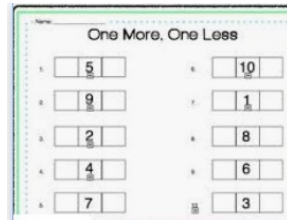
Subtraction




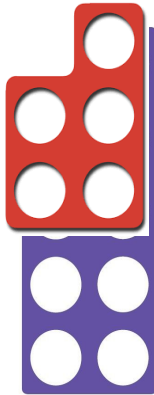


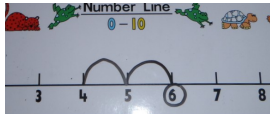
Counting, addition and subtraction should work alongside one another. For example, when teaching more and less this should be done through counting on a number line forwards and backwards, adding one more and subtracting one more. Ideas have been presented for moving from concrete to abstract understanding. However, there will be some of our pupils who will always work in the concrete and pictorial stages. Moving through these stages will develop fluency and generalisation as pupils are exposed to the same skill in different contexts and with different resources.

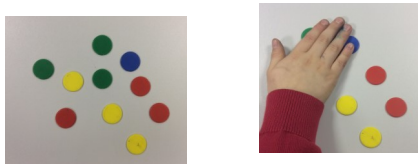

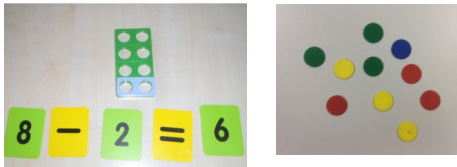
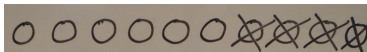
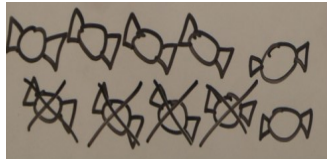
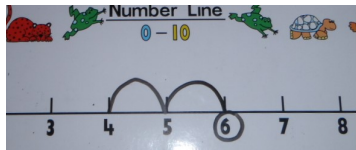
Concrete – students should have the opportunity to use objects and manipulatives to help them understand what they are doing.

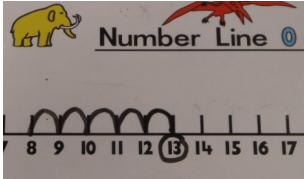
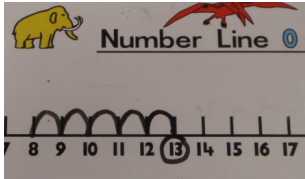
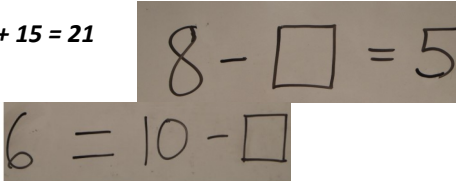
Pictorial – students should then build on this concrete approach by using pictures. This can then be used to reason and solve problems.

Abstract – secure students should be able to use numbers and key concepts with confidence.

Skill	Concrete	Pictorial	Abstract
<p>Understanding of more and less.</p> <p><i>e.g. indicating which plate has less biscuits on it.</i></p> <p>(refer to addition policy and link with concept of more)</p>	<p>Practical situations, 'who has less sweets?' Comparing counting bears, cubes etc</p> 		
<p>Pupil can compare two given numbers of objects saying which is more and which is less.</p> <p>(link to addition policy)</p>			
<p>Understand concept of adding one and subtracting one.</p>	<p>Objects 'give me one more' 'take one away' practical situations e.g building blocks, snack time,</p> 		

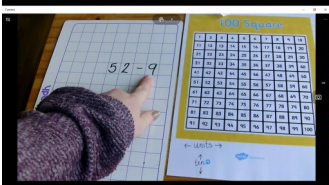
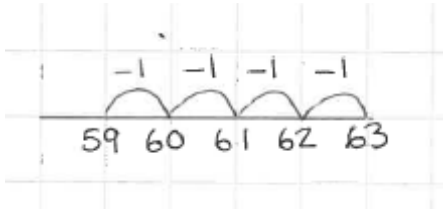
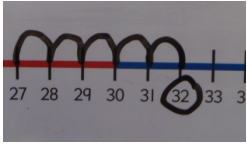
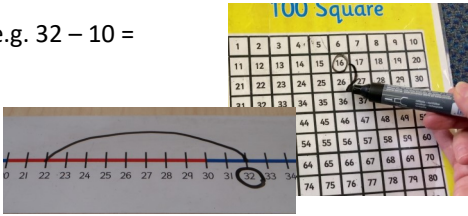
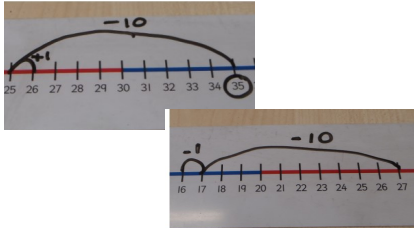
Skill	Concrete	Pictorial	Abstract
<p>Understanding that subtracting a number of objects from the larger amount makes less.</p> <p><i>Start with 5 apples. Remove 2. "More or less?"</i></p>	 <p>Here are 7 cubes. Take away 3 cubes.</p> 		
<p>Know subtraction number facts (link to number bonds addition policy)</p> <p>5-3=2</p> <p>10-6 = 4</p>			
<p>Begin to understand subtraction as taking away using sets of objects up to 10 to model simple number stories.</p> <p><i>e.g. Count 4 pennies into a purse. Stake 2 pennies out. How many pennies are left?</i></p> <p><i>Put 8 biscuits on a plate. Can the child take 5 away and work out how many are left?</i></p> <p>e.g. 4 frogs sat on a log. 2 jumped off. How many left?</p>	<p>Using familiar number songs embed the concept of takeaway using props.</p> 		<p>Subtraction by overlaying and covering up</p> <p>E.g.</p> <p>9-5 =</p> 
<p>Understanding '-' means to takeaway</p> <p>Understanding = means equal amount e.g. 10-3 is the same as 7. (this should be embedded from addition)</p>		<p>Dinosaur Subtraction</p> <p>Write the answers in the circles.</p> 	<p>Using number lines will support abstract development in understanding subtraction. Only introduce number lines when a child is confident using objects and pictures</p> 

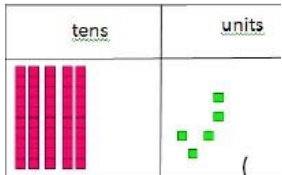
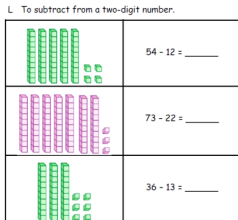
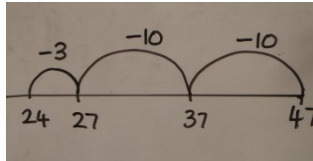
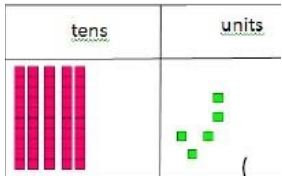
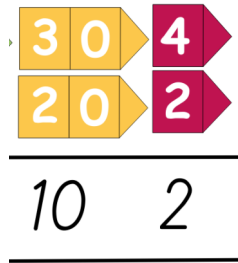
Skill	Concrete	Pictorial	Abstract
<p>Pupil can use the language of: equal to, more than, less than (fewer), most, least.</p> <p>Given a number, pupil can identify one more and one less.</p>	<p>e.g. Using different representations of numbers (counters, objects, straws – bundles of tens and ones, base 10 apparatus, place value cards) pick two different numbers between 0 and 30 and compare two numbers and representations using the language of: equal to, more than, less than (fewer), most and least.</p> <p>What number is one more than 7, 15, 24? What number is one less than 6, 17 and 29?</p>		
<p>Solve simple problems using objects and or marks/pictures to represent.</p> <p>E.g. Sam had 10 sweets. He gave away 4. How many were left?</p>	<p>Solve problems using practical apparatus.</p> <p>$10 - 6 = 4$</p>  <p>Use fingers to take away numbers e.g. $8 - 2$</p>  <p>Use practical objects to subtract numbers higher than 10.</p> 	<p>Use pictures to represent subtracting larger numbers e.g. $10 - 4$</p>  	<p>Use numeral cards to label sets of objects.</p> <p>Use the – symbol.</p> <p>e.g. $8 - 2 =$</p>
<p>Develop understanding of number lines to solve subtraction below 10.</p>			<p>Count back in ones on a marked number track within 10.</p> <p>e.g. $6 - 2 =$</p> 

Skill	Concrete	Pictorial	Abstract
Solve subtraction problems involving bridging 10	$13 - 5 =$ $16 - 9 =$ Removing objects from a line		Using the number line to solve problems involving bridging 10. e.g. $13 - 5 =$ 
Solve calculations involving missing numbers. E.g. $8 - \square = 5$ <i>Link to addition policy and understanding of inverse operations</i>		Use a number line to find the missing number 	Know that to find the missing number in $8 - ?? = 5$ You can do $8 - 5 = 3$ Or to find the first missing number, which is always more (link to prior learning about more/less), you add $?? - 6 = 15$ $6 + 15 = 21$ 
Know subtraction facts to 20 (link to addition facts)			Encourage checking strategies such as counting on.

Skill	Concrete	Pictorial	Abstract
Pupil can read, write and interpret mathematical statements involving addition (-), subtraction (-) and equals (=) signs.			
Understanding '-' means to take away	<i>e.g. Using concrete objects and pictorial representations e.g.</i> <i>counters, cubes, bead strings, ten frames, number lines, pupil works out the following calculations:</i> $17 - 3 =$ $13 + 5 =$ $10 + 7 =$ $12 - 4 =$ $7 + 5 =$ $9 + 7 =$ $6 + 8 =$		Using number lines will support abstract development in understanding subtraction. Only introduce number lines when a child is confident subtracting from groups of objects and pictures
Understanding = means equal amount e.g. 4 and 5 together is the same as 9. Use - and = symbols in simple number sentences. e.g. $6 - 3 =$			
These concepts run across the introduction of formal symbols in addition and subtraction e.g. Represent addition and subtraction examples using number sentences and vice-versa. I have thirteen apples and give 6 away to my friends. How many apples do I have now? How could you write this as a number sentence? You must introduce a range of vocabulary involved in addition and subtraction as below. e.g. Including 'sum', 'add', 'plus', 'total', 'one more', 'two more', 'altogether', 'minus', 'take', 'how many have gone?', 'one less', 'two less', 'leaves', 'how many are left...?			
Pupil can find 1 or 10 more or less than a given number. Pupil can describe and extend simple sequences involving counting on or back in different steps.	<i>e.g. Find 1 more and 1 less and then 10 more and 10 less of the following numbers:</i> $13, 36, 52, 64, 71, 99, 102$ <i>Use number lines and 100 squares to support</i>		<i>Write the next numbers in these sequences:</i> $10, 15, 20, 25, _, _$ $14, 12, 10, 8, _, _$ $0, 3, 6, 9, 12, _, _$ $63, 73, 83, 93, _, _$ $107, 97, 87, 77, _, _$

Skill	Concrete	Pictorial	Abstract
Pupil can read, write and interpret mathematical statements involving addition (-), subtraction (-) and equals (=) signs.			
Pupil can subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations)	<i>e.g. Using concrete objects and pictorial representations e.g. counters, cubes, bead strings, ten frames, number lines, pupil works out calculations:</i>		
<p>Pupil recognises and uses the inverse relationship between addition and subtraction and uses this to check calculations and solve missing number problems (following pages).</p> <p>(Link to subtraction policy—pupils must have a solid grasp of addition and subtraction to understand inverse operations with addition and subtraction)</p>		<p>Use a number line to complete addition calculations both ways and understand they get the same answer</p> <p>Match number sentences—I taught this using an inverse operations monster who like to eat number sentences and then spit them out in a different order —children would match them coming out—you can do this with an inverse operations machine too</p>	<p><i>e.g. Using the numbers 15, 9 and 6, make related number sentences by using +, - and = signs.</i></p> <p>$9 + 6 = 15$</p> <p>$6 + 9 = 15$</p> <p>$15 - 6 = 9$</p> <p>$15 - 9 = 6$</p>
<p>Pupil can recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.</p> <p><i>e.g. If $2 + 8 = 10$, what is $20 + 80$? What do you need to add to 70 to make 100?</i></p> <p>$80 - 60 = 20 + \square$ $7 + 3 = 6 + \square$</p> <p><i>Write the total:</i></p> <p>$6 + 5 + 5 + 4 =$</p> <p><i>Explain how you worked out the answer.</i></p> <p><i>Warren bought two fruits. He spent 30p altogether. He bought an orange for 21p, what did he pay for the other fruit?</i></p>			

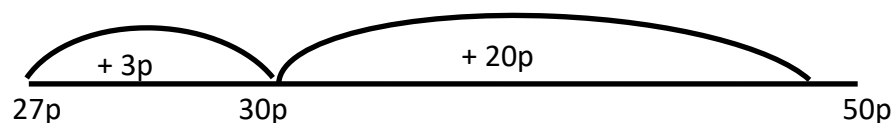
Skill	Concrete	Pictorial	Abstract
Using number lines/100 squares to subtract from a single digit number from a higher number		<p data-bbox="1108 145 1599 209">Use a 100 square to subtract 2 digit by 1 digit e.g. $63 - 4 =$</p> 	<p data-bbox="1630 145 2121 209">Use a number line to subtract 2 digit by 1 digit e.g. $63 - 4 =$</p> 
Using number lines to jump in ones/tens from higher numbers.		<p data-bbox="1108 483 1599 547">Use a marked number line to count back in jumps of ones. e.g. $32 - 5 =$</p>  <p data-bbox="1108 722 1599 786">Use a marked number line/100 square to count back in jumps of tens. e.g. $32 - 10 =$</p>  <p data-bbox="1108 1018 1599 1098">Use a marked number line/100 square. e.g. $35 - 9$ and $27 - 11$</p> 	

Skill	Concrete	Pictorial	Abstract												
<p>Develop an understanding of subtracting 2 digit numbers using equipment and then with empty number lines.</p> <p><i>Link to counting policy—children must be confident counting forwards and backwards in tens, as well as place value, to master this skill</i></p>	<p>55—32 Count out the larger number Take away 2 ones Take away 3 tens Count what is left</p> 	<p>Refer back to work using marked number lines and squares if needed before moving to unmarked.</p> 	<p>Use an empty number line to subtract pairs of two digit numbers in jumps of ones and tens.</p> <p>e.g. $47 - 23 =$</p> 												
<p>Beginning column subtraction</p>	<p>Use dienes equipment as previous page but set out the calculation vertically</p> <p>55 -32</p> 	<p>Expanded column subtraction using place value cards</p> 	<p>Progress to written method of expanded column subtraction</p> <table><tr><td></td><td>Tens</td><td>Ones</td></tr><tr><td>35</td><td>=30</td><td>5</td></tr><tr><td>-23</td><td>=20</td><td>3</td></tr><tr><td colspan="3"><hr/></td></tr></table>		Tens	Ones	35	=30	5	-23	=20	3	<hr/>		
	Tens	Ones													
35	=30	5													
-23	=20	3													
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Teaching finding change (*See Measures—Money & Time policy*)

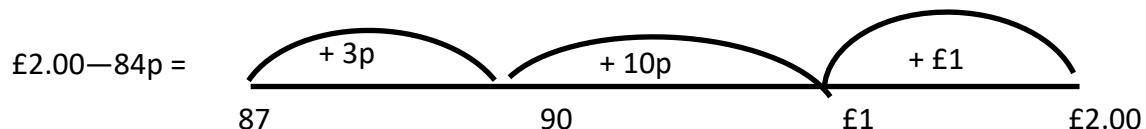
Think about heading to the shop—when you are working out your change—I bet you automatically count up from the amount you are spending to the amount you have handed over. For example, you take 50p and spend 27p. I would add 3 to get to 30 and then 20 to get to 50 giving me a total of 23p change (I understand you probably don't need to do any calculation to work this out personally—this is to illustrate). We can use an empty number line to teach children to count on to find change

$$50\text{p} - 27\text{p} = 23\text{p}$$



$$20 + 3 = 23$$

It works with pounds and pence too—you will need to adjust the process criteria



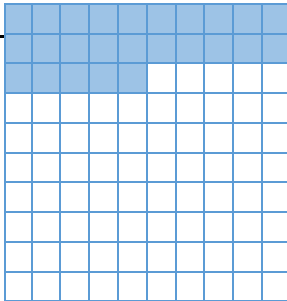
$$£1 + 10 + 3 = £1.13$$


Process criteria (for money within £1)

- Write the smaller number at the beginning of the number line
- Write the larger number at the end of the number line
- Count on in ones until you get to the next ten
- Count on in tens until you get to the final number
- Add together the numbers in the jumps
- Go back and record your answer at the end of the number sentence

Checking

At each stage it is important to encourage children to check answers using alternate methods e.g. counting on or using inverse operations

Skill	Concrete	Pictorial																	
<p>Pupil can subtract numbers mentally, including: -</p> <ul style="list-style-type: none">a three-digit number and onesa three-digit number and tensa three digit number and hundreds <p>(Dependent on concrete understanding of place value)</p>	<p>$163 - 2$</p> <p>$278 - 6$</p> <p>$152 - 40$</p> <p>$478 - 50$</p> <p>$517 - 300$</p> <p>$133 - 30$</p> <p>$60 + 50 - 40$</p>																		
<p>Pupil can subtract numbers with up to three digits, using formal written methods of columnar subtraction</p>	<p>e.g. Here are some cards with numbers on them:</p> <table><tr><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td></tr></table> <p>Use these five number cards to make this calculation correct.</p> <table><tr><td></td><td></td><td></td></tr><tr><td>-</td><td></td><td></td></tr><tr><td colspan="3"><hr/></td></tr><tr><td>5</td><td>1</td><td>3</td></tr></table>	5	4	3	2	1				-			<hr/>			5	1	3	
5	4	3	2	1															
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5	1	3																	
<p>Pupil recalls and uses addition and subtraction facts for 100 (multiples of 5 and 10).</p>	<p>$100 - 20 = \underline{\hspace{1cm}}$</p> <p>$100 - 75 = \underline{\hspace{1cm}}$</p> <p>$100 - 97 = \underline{\hspace{1cm}}$</p> <p>$200 - 150 = \underline{\hspace{1cm}}$</p>	<p>Use 100 squares</p> <p>E.g. $100 - 25$ counting remaining squares</p> 																	
<p>Pupil adds amounts of money to give change, using both £ and p in practical contexts.</p> <p>(see money policy)</p>	<p>Inverse of counting on to make target total</p> <p>Cost 20p pay with £1: $20p + 80p = £1$ so $£1 - 20p = 80p$</p> <p>Cost 65p pay with £1: $35p + 65p = £1$ so $£1 - 65p = 35p$</p>																		

Skill	Concrete	Pictorial																				
Pupil can subtract mentally combinations of two and three digit numbers and decimals to 1 decimal place.	$380-60 = \underline{\hspace{1cm}}$ $380-45 = \underline{\hspace{1cm}}$ $170-90 = \underline{\hspace{1cm}}$ $25.5-12.5 = \underline{\hspace{1cm}}$ $25-12.5 = \underline{\hspace{1cm}}$																					
Pupil can recall and use subtraction facts for 100 and for multiples of 100 totalling 1000.	$100-[\quad] = 30$ $200-[\quad] = 140$ $300-[\quad] = 90$ $1000-[\quad] = 400$	Practical illustration of subtraction using Diens 100-squares to 1000- block 																				
Pupil can subtract numbers with up to 4 digits and decimals with one decimal place using the formal written methods of columnar subtraction where appropriate.	<div><div>-</div><table><tr><td>2</td><td>6</td><td>3</td><td>.</td><td>5</td></tr><tr><td></td><td>4</td><td>5</td><td>.</td><td>4</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td>.</td><td></td></tr></table><div>Use Hundreds, Tens, Units headings</div></div>	2	6	3	.	5		4	5	.	4									.		
2	6	3	.	5																		
	4	5	.	4																		
			.																			

Common misconceptions when subtracting

- Taking the smallest digit from the biggest in a 2 or 3 digit calculation, regardless of the full number e.g.

$$\begin{array}{r}
 356 \\
 - 174 \\
 \hline
 222
 \end{array}$$

Thinks
 $5 - 7 = 2$

Common misconceptions when subtracting

- Justifying column subtraction with 2 & 3 digit numbers to the left
-
- Failure to keep decimal points in line (including in the answer)

$$\begin{array}{r} 345 \\ - 32 \\ \hline 25 \end{array} \quad \times$$

$$\begin{array}{r} 594 \\ - 16.2 \\ \hline 43.2 \end{array} \quad \times$$

$$\begin{array}{r} 62.4 \\ - 26.5 \\ \hline 35.9 \end{array} \quad \times$$