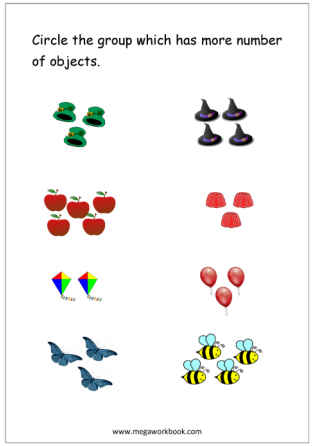
Addition

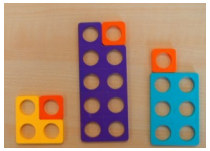
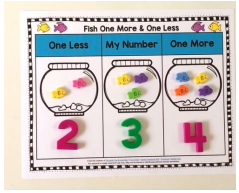
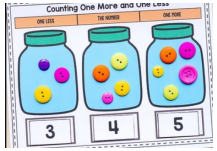
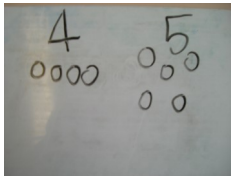
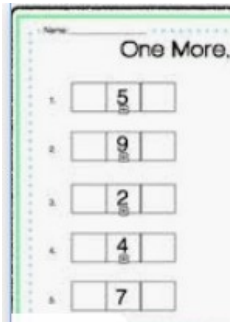
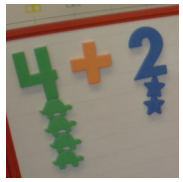
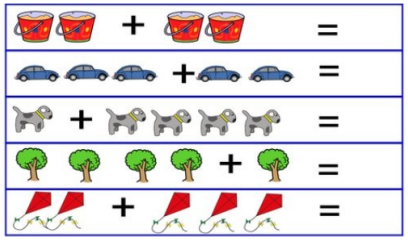
Counting, addition and subtraction should work alongside one another. For example, when teaching more and less this should be done through counting on a number line forwards and backwards, adding one more and subtracting one more. Ideas have been presented for moving from concrete to abstract understanding . However, there will be some of our pupils who will always work in the concrete and pictorial stages. Moving through these stages will develop fluency and generalisation as pupils are exposed to the same skill in different contexts and with different resources.

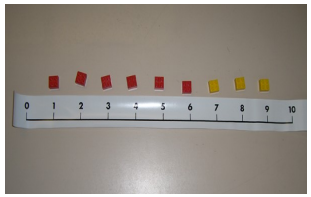
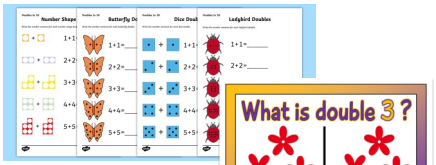
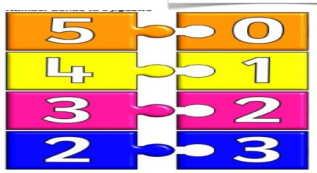
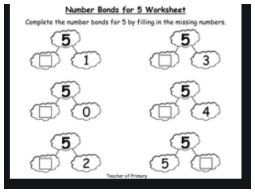
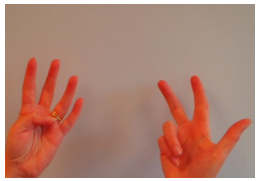
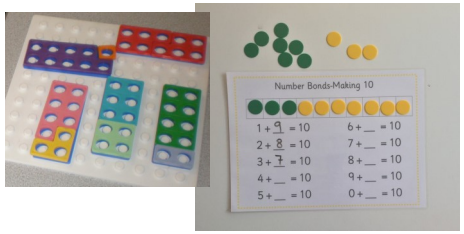
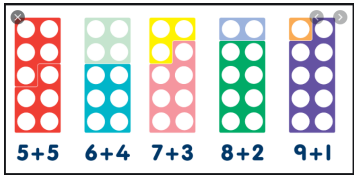
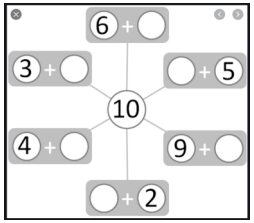
Concrete – students should have the opportunity to use objects and manipulatives to help them understand what they are doing.

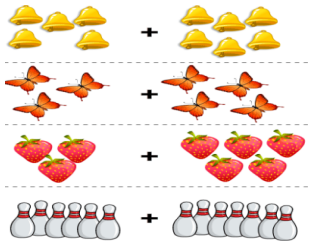
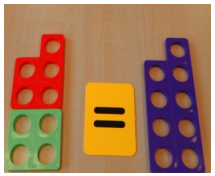
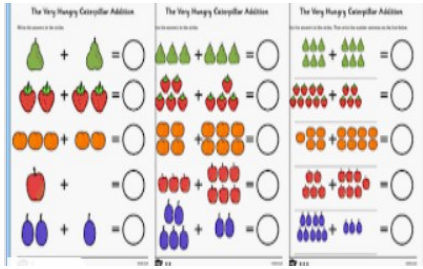
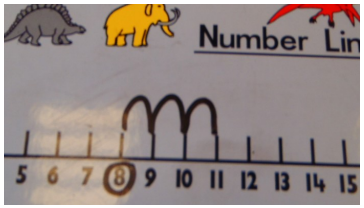
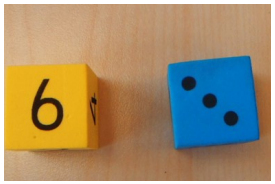
Pictorial – students should then build on this concrete approach by using pictures. This can then be used to reason and solve problems.

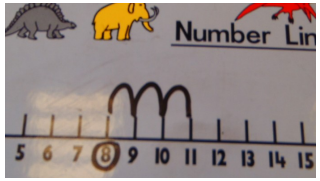
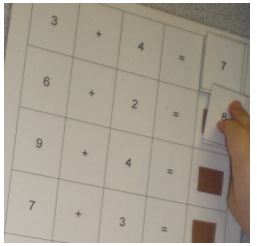
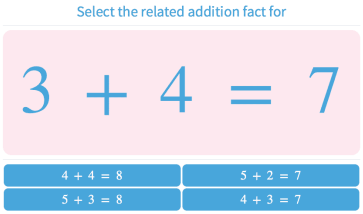
Abstract – secure students should be able to use numbers and key concepts with confidence.

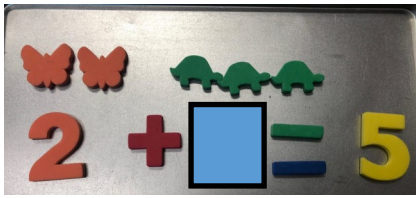
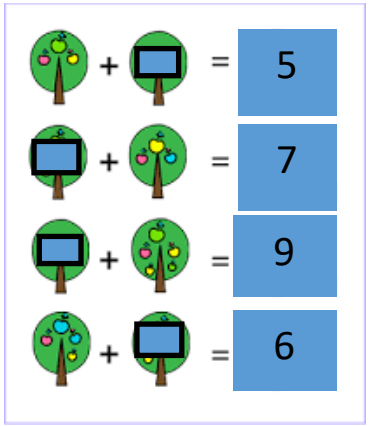
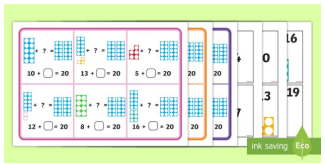
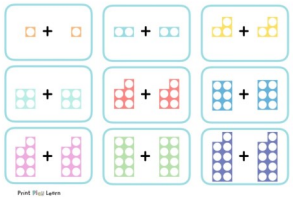
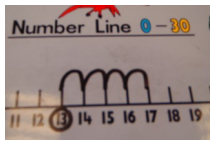
Skill	Concrete	Pictorial	Abstract
Demonstrate awareness of contrasting quantities where there is a marked difference,	e.g. one cake and lots of cakes on plates		
Pupil can use 'one to one' correspondence when pairing objects,	e.g. put a straw in each carton of milk, put a spoon in each cup		
Pupil can demonstrate an understanding of the concept of more,	e.g. a child giving out cups has not got one for each child and indicates they need 'more'.		
Understanding of more and less in groups - can compare two given numbers of objects saying which is more and which is less. Children need progressive experiences where they can compare collections and begin to talk about which group has more things. Initially, the groups need to be very obviously different, with one group having a widely different number of things. Collections should also offer challenges, such as including more small things and fewer large things, to draw attention to the numerosity of the comparison, i.e. the number of things, not the size of them.	Ask which pile of toys, stickers etc a child wants  Children need the opportunity to see that groups could consist of equal numbers of things. Children can check that groups are equal, by matching objects on a one-to-one basis.		Children can compare numbers that are far apart, near to and next to each other. For example, 8 is a lot bigger than 2 but 3 is only a little bit bigger than 2.

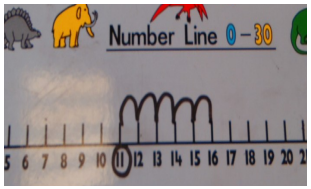
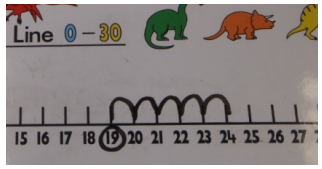
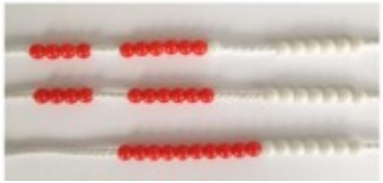
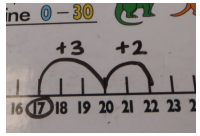
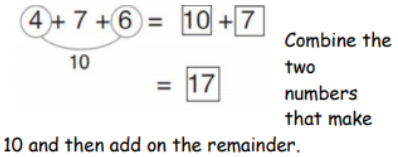
Skill	Concrete	Pictorial	Abstract
Understand concept of one more/adding one up to 10 This should be linked to the counting policy—when counting along a number line, talk about one more than, one less than	In practical situations add one to and take one away from a number of objects (up to 10) then say or sign how many there are now. *e.g. add one more sweet to the dish.*    	 	
Understanding the function of addition- combining amounts to find the total of two or more sets of objects to find the total (to 10) Can add numbers of objects to 10. *e.g. Find all the dominoes that have a total of 7.* *Show me 3 fingers on one hand and 4 fingers on the other.* *How many are you showing altogether?* *Can you give me another way of showing 7?*	   		

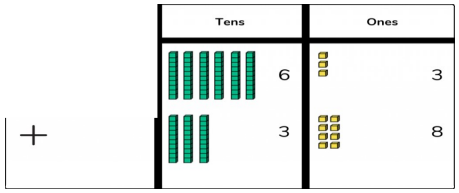
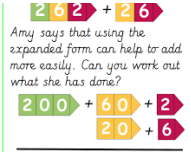
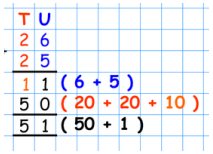
Skill	Concrete	Pictorial	Abstract
Count the sets of objects on a number track/line			
Knows some simple addition facts Start to know doubles to 10 *e.g. Double 4, 1, 3, perhaps using fingers to support.* Start to know number bonds to 5 (link to page 2 here and subtraction policy) *e.g. partitioning the number 5 as 0+5, 1+4 etc*	 Use practical activities to show how to double a number. 	 	
Begin to use fingers to represent two sets and add together			
Knows some simple addition facts Move onto number bonds to 10 *Use fingers to support knowledge of number bonds for number 10.*			

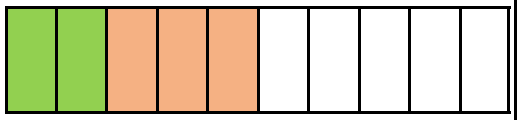
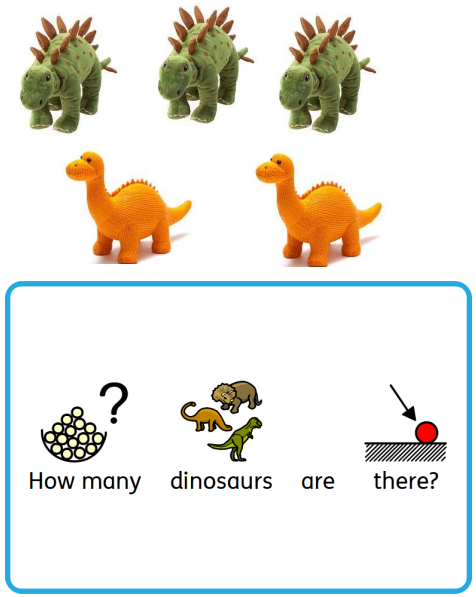
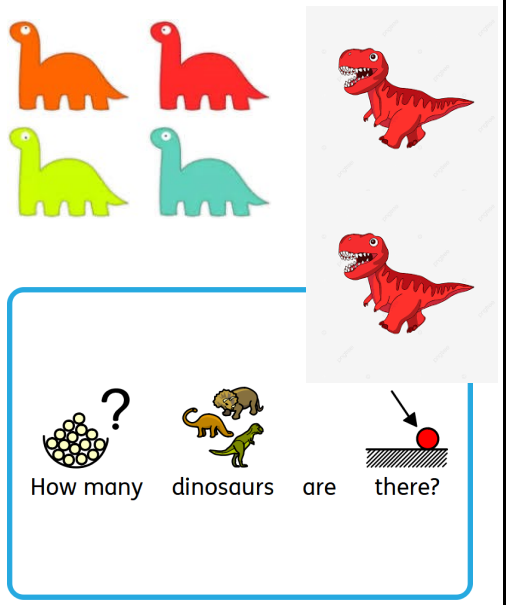
Skill	Concrete	Pictorial	Abstract
Pupil can read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.			
Understanding '+' means to add.			Using number lines will support abstract development in understanding addition. Only introduce number lines when a child is confident counting groups of objects and pictures
Understanding = means equal amount e.g. 4 and 5 together is the same as 9. Use + and = symbols in simple number sentences. e.g. 6 + 3 =			
These concepts run across the introduction of formal symbols in addition and subtraction **e.g. Represent addition and subtraction examples using number sentences and vice-versa. I have thirteen apples and give 6 away to my friends. How many apples do I have now? How could you write this as a number sentence?** **You must introduce a range of vocabulary involved in addition and subtraction as below.** **e.g. Including 'sum', 'add', 'plus', 'total', 'one more', 'two more', 'altogether', 'minus', 'take', 'how many have gone?', 'one less', 'two less', 'leaves', 'how many are left...?**			
Hold the largest number in head and count on the smaller number with fingers/ count on when one group of objects is hidden			5+12 = 17 No matter the order, start at the larger number and count on using fingers to support if needed If able, use number lines and start with the larger number.

Skill	Concrete	Pictorial	Abstract
Use a marked number line to 10, then 20 to count on in ones illustrating and making number stories/sentences e.g. add 11 pencils to 5 pencils e.g. 8 + 3 =			
Understand that addition can be done in any order—commutative (but subtraction cannot)	Whatever the order, remind children about prior learning and starting with the larger number	Using objects understand that 2+3 is the same as 3+2	
Pupil recognises and uses the inverse relationship between addition and subtraction and uses this to check calculations and solve missing number problems (following pages). (Link to subtraction policy—pupils must have a solid grasp of addition and subtraction to understand inverse operations with addition and subtraction)		Use a number line to complete addition calculations both ways and understand they get the same answer Match number sentences—I taught this using an inverse operations monster who like to eat number sentences and then spit them out in a different order —children would match them coming out—you can do this with an inverse operations machine too	*e.g. Using the numbers 15, 9 and 6, make related number sentences by using +, - and = signs.* 9 + 6 = 15 6 + 9 = 15 15—6 = 5 15—5 = 6
Pupil represents and uses number bonds and related subtraction facts within 20.	*e.g. How many different ways can you show the 12 spots on this ladybird?* *I have a 20p coin and buy a pencil for 12p. What change will I get?* *I have 9 stickers and need 15 before I can get a head teacher's award. How many more stickers do I need?*		

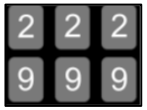
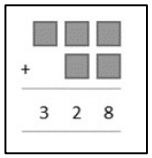
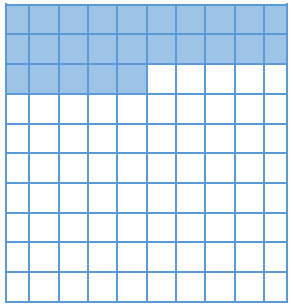
Skill	Concrete	Pictorial	Abstract
Find the missing number when the second number is hidden. 3 + ☐ = 7 Find the missing number when the first number is hidden. ☐ + 5 = 8 Initially use equipment then fingers to count on for missing numbers.	Begin to understand that you can use the equipment to count on, start at two and add on the three additional objects while counting 3,4,5  	 	Understand that to find the missing number below—you can use the inverse 3 + ☐ = 7 7—3 = 4 ☐ + 5 = 8 8—5 = 3
Pupil can recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100. (link to subtraction policy) e.g. *If 2 + 8 = 10, what is 20 + 80? What do you need to add to 70 to make 100?* *80 – 60 = 20 +* ☐ *7 + 3 = 6 +* ☐ *Write the total:* *6 + 5 + 5 + 4 =* *Explain how you worked out the answer.* *Warren bought two fruits. He spent 30p altogether. He bought an orange for 21p, what did he pay for the other fruit?*	Use range of resources to ensure children are fluent with number facts to 5, 10 and 20. Link this with facts to 100. Include doubles and halves 	Use pictorial representations to recall number bonds and facts 	Understand if: 2+2=4, 20+20 = 40 3+7 = 10, 30+70 = 100 Double 6 is 12, so half of 12 is 6
Pupil can add one-digit and two-digit numbers to 20, including zero *(using concrete objects and pictorial representations)*	*e.g. Using concrete objects and pictorial representations e.g.* *counters, cubes, bead strings, ten frames, number lines, pupil works out the following calculations:* *13 + 5 =* *10 + 7 =* *7 + 5 =* *9 + 7 =* *6 + 8 =*	Put biggest number first when adding two numbers and use marked number lines to model this. E.g. re-order 4 + 13 = to 13 + 4 =  	

Skill	Concrete	Pictorial	Abstract
Apply to more complex missing numbers: 16 = 11 + ☐ ☐ + 7 = 20 9 + 3 = ☐ + 2	Pupil can solve one-step problems that involve addition , using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ - 9. *e.g. Work out the missing numbers in the following number sentences. Can you 'tell a story' to match the number sentences?* *6 + ☐ + △ = 14*	16 = 11 + ☐ 	
Use a marked number line bridging through ten when adding single digit numbers, e.g. 19 + 5 =			
Add 3 single digit numbers bridging ten by counting or using a marked number line. 7 + 3 + 2	4+6+7—combine the 4 and 6 to make 10 and then add on the 7 		17+3+2 Know that 17 +3 = 20 and then add the 2 
Pupil can make links between operations and number facts to allow them to generalise			e.g. *If 2 + 8 = 10, what is 20 + 80? What do you need to add to 70 to make 100?* *80 – 60 = 20 +* ☐ *7 + 3 = 6 +* ☐ *Write the total:* *6 + 5 + 5 + 4 =* *Explain how you worked out the answer.*

Skill	Concrete	Pictorial	Abstract
Pupil can add numbers using concrete objects, pictorial representations, and mentally, including: - • a two-digit number and tens	This can all be completed using dienes or numicon. Large dienes are particularly supportive to understand this concept before moving to abstract column addition		
Pupil can add numbers using concrete objects, pictorial representations, and mentally, including: - • two two-digit numbers ***See final column for steps through formal addition methods. Only move onto the next stage when a child demonstrates depth of understanding over time.*** ***If the next stage is challenging, move back a step and reinforce or use concrete dienes equipment to scaffold.***	Dienes equipment can be used at every stage to demonstrate each written method to support children to understand abstract methods 	Use visual representations of equipment. You can line the equipment vertically on a grid, in the same way as column addition rather than always horizontally (63+38) 	Begin with expanded method no carrying 63 = 60 + 3 +32 = 30 + 2 95 = 90 + 5  Introduce carrying (use dienes to support) 49 = 40 + 9 +24 = 20 + 4 73 =60 + 13 = 60 + 10 + 3 Move to more complex expanded addition method  Finally move to compact column addition 
This takes us to the end of PIVATS milestone 2 NEED TO ADD MILESTONE 3 AND 4			

Skill	Concrete	Pictorial	Abstract
Add decimal fractions including money *(Refer to Fractions Policy)*		Colouring in sections of a 10-grid E.g. 0.2 + 0.3 	Column addition with decimal points in line 0.2 + <u>0.4</u>
Interpret and solve simple addition word problems			

Common misconceptions when adding
• Counting on including the starting number (e.g. 4+3 : “4,5,6”) • Over-reliance on concrete examples

Skill	Concrete	Pictorial
Pupil can add and subtract numbers mentally, including: - • a three-digit number and ones • a three-digit number and tens • a three digit number and hundreds (Dependent on concrete understanding of place value)	*163 + 7* *278 + 6* *152 + 40* *478 + 50* *517 + 300* *133 + 40* *60 + 50 + 40*	
Pupil can add numbers with up to three digits, using formal written methods of columnar addition	*e.g. Here are some cards with numbers on them:*  *Use five of the number cards to make this calculation correct.* 	
Pupil recalls and uses addition facts for 100 (multiples of 5 and 10).	40 + [ ] = 100 [ ] + 75 = 100 110 + [ ] = 200 275 + [ ] = 300	Use 100 squares E.g. 25 + [ ] = 100  Use Diens 10s, 5s and units to match 100 square 
Pupil adds amounts of money to give change, using both £ and p in practical contexts. (see money policy)	50p = 20p + _____ £1 = 70p + _____ £1 = 85p + ___ + ___	85p + ☐ =  Draw coins

Skill	Concrete	Pictorial
Pupil can add mentally combinations of two and three digit numbers and decimals to 1 decimal place.	220 + 300 = _____ 450 + 45 = _____ 68 + 102 = _____ 12 + 5.6 = _____ 41 + 22.5 = _____	
Pupil can recall and use addition facts for 100 and for multiples of 100 totalling 1000.	200 + [ ] = 1000 700 + [ ] = 1000 [ ] + 100 = 1000	Practical addition of Diens 100-squares to 1000– block 
Pupil can add numbers with up to 4 digits and decimals with one decimal place using the formal written methods of columnar addition where appropriate.	+ <table><tr><td></td><td>2</td><td>3</td><td>.</td><td>5</td></tr><tr><td>1</td><td>8</td><td>5</td><td>.</td><td>4</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td>.</td><td></td></tr></table>	

Common misconceptions when adding

- Justifying column addition with 2 & 3 digit numbers to the left

```
 345
+ 32
 665  ✗
```

-

- Failure to keep decimal points in line (including in the answer)

```
  24
+ 16.5
  14.9  ✗
```

```
  22.4
+ 16.5
  38 9  ✗
```